Helical Magnetic Fields from Inflation

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We analyze the generation of seed magnetic fields during de Sitter inflation considering a non-invariant conformal term in the electromagnetic Lagrangian of the form $-\frac{1}{4}I(\phi)F_{\mu\nu}\widetilde{F}^{\mu\nu}$, where $I(\phi)$ is a pseudoscalar function of a non-trivial background field ϕ. In particular, we consider a toy model, that could be realized owing to the coupling between the photon and either a (tachyonic) massive pseudoscalar field and a massless pseudoscalar field non-minimally coupled to gravity, where I follows a simple power-law behavior $I(k,\eta) = g/(-k\eta)^\beta$ during inflation, while it is negligibly small subsequently. Here, g is a positive dimensionless constant, k the wavenumber, η the conformal time, and β a real positive number. We find that only when $\beta = 1$ and $0.1 \lesssim g \lesssim 2$ astrophysically interesting fields can be produced as excitation of the vacuum, and that they are maximally helical.

I. INTRODUCTION

It is not excluded that large-scale, microgauss magnetic fields detected in any type of galaxies have a primordial origin [1, 2]. If so, they have probably been generated during an inflationary epoch of the universe, since in this case their correlation length can be as large as the galactic one. (For possible generating mechanisms see, e.g., Ref. [3, 4, 5, 6].) An overwhelming proof of "primordial origin" of galactic fields could come from the observation of peculiar imprints they leave on the Cosmic Microwave Background (CMB) anisotropies [7].

Two considerations are in order. Firstly, to explain galactic magnetism it suffices to generate "seed" magnetic fields prior to galaxy formation of intensity generally much less than 1μG. In fact, due to magnetohydrodynamic turbulence effects and differential rotation of galaxy, extremely week fields can be exponentially amplified. This mechanism, know as "galactic dynamo" [2, 8], successfully explains the main characteristics of observed fields provided that their intensity and correlation length are $B \gtrsim 10^{-33}$G and $\lambda \gtrsim 10$kpc. If dynamo is inefficient, instead, a stronger field, correlated on comoving scales of order 1Mpc, is needed to explain galactic magnetism. In this case the amplification of a primordial seed field is just due to magnetic flux conservation during protogalaxy collapses. Estimates based on "spherical infall model" indicate that the time when protogalaxy collapses begins corresponds to a redshift $z_{\rm pg}$ not greater than 50 [1], although galactic disks are assembled at a much later epoch, corresponding to redshifts of order $z \sim$ few. Taking the "less conservative" value $z_{\rm pg} \simeq 50$, one can shows that a comoving field $B \gtrsim 10^{-14}$G explain magnetization of galaxies (see, e.g., Ref. [6]).

Secondly, any viable mechanism of generation must repose on the breaking of conformal invariance of standard electrodynamics, otherwise the produced fields are vanishingly small [3].

In this paper, we study the possibility to generate seed magnetic fields during inflation in a non-invariant conformal theory of electromagnetism described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\, I F_{\mu\nu}\widetilde{F}^{\mu\nu}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor and $\widetilde{F}_{\mu\nu} = (1/2\sqrt{-g}\,)\, \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ its dual, with g the determinant of the metric tensor and $\epsilon_{\mu\nu\rho\sigma}$ the Levi-Civita tensor. The quantity I is assumed to be a function of a non-trivial background pseudoscalar field ϕ and its actual form will be specified later.

In the seminal paper by Turner and Widrow [3], it was suggested that, during inflation, small fluctuating magnetic fields could be amplified due to the coupling with the axion field (in this case, the pseudoscalar I is $I = g_{a\gamma}\phi$, where ϕ is the axion field and $g_{a\gamma}$ is the axion-photon coupling constant). This possibility was exhaustively studied in the subsequent papers by Garretson, Field and Carroll [9], and Field and Carroll [10]. A different scenario was analyzed by Prokopec in Ref. [11] where the pseudoscalar ϕ was supposed to drive (chaotic or extended) inflation. As a matter of fact, in that analysis I was taken to be a uniform and slowly varying function of time during inflation and zero subsequently. In all cases, however, it was shown that no interesting cosmic fields can be generated.

An interaction term of the form $IF_{\mu\nu}\widetilde{F}^{\mu\nu}$ is responsible for the creation of magnetic fields described by an "unbalanced" superposition of left- and right-handed photons. In general, a time variation of I produces a change of $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ which can be quantified by [12] $\frac{1}{4V}\int_{t_1}^{t_2} d^4x F_{\mu\nu}\widetilde{F}^{\mu\nu} = H_V(t_2) - H_V(t_1)$. The quantity H_V is the so-called magnetic helicity density (in the volume V) and is proportional to the difference between the number of left- and right-handed photons (see Section III). It is defined by

$$H_V = \frac{1}{V}\int_V d^3x \mathbf{A}\cdot\mathbf{B}, \tag{2}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the vector potential and the magnetic field, respectively. In magneto-hydrodynamics [13], a non-vanishing magnetic helicity indicates a non-trivial configuration of magnetic flux tubes (which are twisted or linked). In particle physics, magnetic helicity is known as the Abelian, Euclidean Chern-Simons term. As shown by Jackiw and Pi [14], a helical magnetic field results from the projection of a non-Abelian gauge field onto a fixed direction in isospace. The magnetic helicity coincides then with the winding number carried by the non-Abelian vacuum configuration.

It is worth noting that, since the magnetic helicity is odd under discrete P and CP transformations, if detected on cosmic scales it would indicate a macroscopic P and CP violation. This peculiar characteristic of helical magnetic fields is very attractive and, in fact, has induced many authors to devise models of generation of primordial helical fields in the last years [3, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Among the many mechanisms proposed, a very intriguing one is that of Cornwall [16] and Vachaspati [19] (see also Ref. [23, 24, 25]) according to which a net helicity $H_V \sim -n_b/\alpha$ [19] is generated as a by-product of baryon-number-violating processes taking place during electroweak baryogenesis,

where n_b is the present cosmological baryon number density and α the fine structure constant. Interestingly, as it has been pointed out by Giovannini [26], the reverse it also true: decaying non-trivial configuration of (hyper-)magnetic flux tubes can seed the Baryon Asymmetry of the universe.

The plan of the paper is as follows. In Section II, we consider a toy model which generalize the case analyzed by Prokopec in Ref. [11] and enables helical seed fields to be produced as excitation of the vacuum. In Section III, we compute the amount of magnetic helicity associated to the generated fields. In the Conclusions we summarize our results.

II. CREATION OF HELICAL MAGNETIC FIELDS

a. General Considerations

We assume that during inflation the universe is described by a de Sitter metric $ds^2 = a^2(d\eta^2 - d\mathbf{x}^2)$, where $a(\eta)$ is the expansion parameter, $\eta = -1/(aH)$ is the conformal time, and H is the Hubble parameter. The conformal time is related to the cosmic time t through $d\eta = dt/a$. We normalize the expansion parameter so that at the present time t_0, $a(t_0) = 1$.

We work in the Coulomb gauge, $A_0 = \sum_{i=1}^{3} \partial_i A_i = 0$, and we expand the electromagnetic field $A_\mu = (A_0, \mathbf{A})$ as

$$\mathbf{A}(\eta, \mathbf{x}) = \sum_{\alpha=1,2} \mathbf{A}_\alpha(\eta, \mathbf{x}), \tag{3}$$

$$\mathbf{A}_\alpha(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2k}} \, \boldsymbol{\varepsilon}_{\mathbf{k},\alpha} \, a_{\mathbf{k},\alpha} A_{k,\alpha}(\eta) \, e^{i\mathbf{k}\mathbf{x}} + \text{h.c.}, \tag{4}$$

where $k = |\mathbf{k}|$ and $\boldsymbol{\varepsilon}_{\mathbf{k},\alpha}$ (with $\alpha = 1, 2$) are the transverse polarization vectors satisfying the completeness relation $\sum_\alpha (\varepsilon_{\mathbf{k},\alpha})_i (\varepsilon^*_{\mathbf{k},\alpha})_j = \delta_{ij} - k_i k_j / k^2$. The annihilation and creation operators $a_{\mathbf{k},\alpha}$ and $a^\dagger_{\mathbf{k},\alpha}$ satisfy the usual commutation relations $[a_{\mathbf{k},\alpha}, a^\dagger_{\mathbf{k}',\alpha'}] = (2\pi)^3 \delta_{\alpha\alpha'} \delta(\mathbf{k} - \mathbf{k}')$, $[a_{\mathbf{k},\alpha}, a_{\mathbf{k}',\alpha'}] = [a^\dagger_{\mathbf{k},\alpha}, a^\dagger_{\mathbf{k}',\alpha'}] = 0$, with $a_{\mathbf{k},\alpha}|0\rangle = 0$, where $|0\rangle$ is the vacuum state normalized as $\langle 0|0\rangle = 1$.

Introducing the average magnetic and electric fields on a comoving scale λ as [5]

$$a^2 \mathbf{B}_{\lambda,\alpha}(\eta, \mathbf{x}) = \int d^3y \, W_\lambda(|\mathbf{x} - \mathbf{y}|) \, \nabla \times \mathbf{A}_\alpha(\eta, \mathbf{y}), \tag{5}$$

$$a^2 \mathbf{E}_{\lambda,\alpha}(\eta, \mathbf{x}) = -\int d^3y \, W_\lambda(|\mathbf{x} - \mathbf{y}|) \, \dot{\mathbf{A}}_\alpha(\eta, \mathbf{y}), \tag{6}$$

where $W_\lambda(|\mathbf{x}|) = (2\pi\lambda^2)^{-3/2} e^{-|\mathbf{x}|^2/(2\lambda^2)}$ is a gaussian window function and a dot denotes differentiation with respect to the conformal time, we can define their vacuum expectation values as

$$B^2_{\lambda,\alpha}(\eta) = \langle 0|\, |\mathbf{B}_{\lambda,\alpha}(\eta,\mathbf{x})|^2 |0\rangle, \tag{7}$$

$$E^2_{\lambda,\alpha}(\eta) = \langle 0|\, |\mathbf{E}_{\lambda,\alpha}(\eta,\mathbf{x})|^2 |0\rangle. \tag{8}$$

Taking into account Eqs. (3)-(8), we obtain

$$B^2_{\lambda,\alpha}(\eta) = \int_0^\infty \frac{dk}{k}\, W^2_\lambda(k)\, \mathcal{P}_{k,\alpha}(\eta), \tag{9}$$

$$E^2_{\lambda,\alpha}(\eta) = \int_0^\infty \frac{dk}{k}\, W^2_\lambda(k)\, \mathcal{Q}_{k,\alpha}(\eta), \tag{10}$$

where $W_\lambda(k) = e^{-\lambda^2 k^2/2}$ is the Fourier transform of the window function and

$$\mathcal{P}_{k,\alpha}(\eta) = \frac{k^4}{4\pi^2 a^4}\, |A_{k,\alpha}(\eta)|^2, \tag{11}$$

$$\mathcal{Q}_{k,\alpha}(\eta) = \frac{k^2}{4\pi^2 a^4}\, |\dot{A}_{k,\alpha}(\eta)|^2, \tag{12}$$

are the magnetic and electric power spectra, respectively.

It is worth noting that in any meaningful model, the function $\mathcal{P}_{k,\alpha}/k$ and $\mathcal{Q}_{k,\alpha}/k$ have to be integrable in $k \to 0$ (no infrared divergence) in order to have a finite value for the magnetic and electric fields $B^2_{\lambda,\alpha}$ and $E^2_{\lambda,\alpha}$.

The equation of motion for $\mathbf{A}(\eta,\mathbf{x})$ follows from Lagrangian (1):

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \dot{I}\, \nabla \times \mathbf{A} - \nabla I \times \dot{\mathbf{A}} = 0, \tag{13}$$

It is worth noting that the effect of the last term in the left-hand-side of Eq. (13) is just to cause a precession of $\dot{\mathbf{A}}$ around ∇I. Therefore, its presence does not affect the intensity of $\dot{\mathbf{A}}$ and, in turn, that of $\mathbf{A}$ and that of the related magnetic field. Retaining this term would add a major level of complexity to our analysis without, nevertheless, changing the final result regarding the average intensity of the inflation-produced magnetic field. For this reason, we may write

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \dot{I}\, \nabla \times \mathbf{A} \simeq 0, \tag{14}$$

where the symbol $\simeq$ indicates equality in the sense just discussed.

We are interested in the study of large-scale electromagnetic fields, that is in modes whose physical wavelength is much greater than the Hubble radius, $\lambda_{\rm phys} \gg H^{-1}$ or equivalently $|k\eta| \ll 1$, where $\lambda_{\rm phys} = a\lambda$ and $\lambda = 1/k$ is the comoving wavelength.

If the quantity I is peaked at some small wavelength we expect that, at large scales, the term proportional to I in Eq. (14) is negligible with respect to the second term. In this case we recover the case of free Maxwell theory and no amplification of magnetic modes occurs. Therefore, in the following, we consider a simplified special case where I is different from zero just at large scales. To see how this assumption modifies Eq. (14), it is convenient to work in Fourier space. Inserting Eqs. (3)-(4) in Eq. (14), we get

$$\left(\frac{\ddot{A}_{k,\alpha}}{\sqrt{2k}} + k^2\,\frac{A_{k,\alpha}}{\sqrt{2k}}\right) a_{\mathbf{k},\alpha} + \sum_{\beta=1,2} \int \frac{d^3q}{(2\pi)^3}\, \dot{I}_{\mathbf{k}-\mathbf{q}} \Upsilon_{\mathbf{k},\alpha;\,\mathbf{q},\beta}\, \frac{A_{q,\beta}}{\sqrt{2q}}\, a_{\mathbf{q},\beta} + \text{h.c.} \simeq 0, \tag{15}$$

where $I_{\mathbf{k}}$ is the Fourier transform of I and

$$\Upsilon_{\mathbf{k},\alpha;\,\mathbf{q},\beta} = i\mathbf{q} \times \boldsymbol{\varepsilon}_{\mathbf{q},\beta} \cdot \boldsymbol{\varepsilon}^{*}_{\mathbf{k},\alpha}\,. \tag{16}$$

Multiplying Eq. (15) by $a^{\dagger}_{\mathbf{k}',\alpha'}$ first on the right and then on the left and subtracting the resulting expressions, and integrating in $\mathbf{k}'$ and summing on α' afterwards, we obtain

$$\frac{\ddot{A}_{k,\alpha}}{\sqrt{2k}} + k^2\,\frac{A_{k,\alpha}}{\sqrt{2k}} + \sum_{\beta=1,2} \int \frac{d^3q}{(2\pi)^3}\, \dot{I}_{\mathbf{k}-\mathbf{q}} \Upsilon_{\mathbf{k},\alpha;\,\mathbf{q},\beta}\, \frac{A_{q,\beta}}{\sqrt{2q}} \simeq 0. \tag{17}$$

Since we are assuming that I is different from zero just at large scales ($k \to 0$), we may write

$$I_{\mathbf{k}} = (2\pi)^3 \delta(\mathbf{k}) I_k, \tag{18}$$

where $\delta(\mathbf{k})$ is a function which is "extremely" peaked at small k. In order to simplify calculations, we assume that $\delta(\mathbf{k})$ is indeed the Dirac delta function. In this case, only modes with small wavenumbers are important when considering the function I_k. For this reason, we may expand $I_{\mathbf{k}-\mathbf{q}}$ for small values of $\mathbf{k}$ but, due to the presence of the Dirac delta, we may expand for small values of $\mathbf{q}$ as well:

$$I_{\mathbf{k}-\mathbf{q}} = (2\pi)^3 \delta(\mathbf{k}-\mathbf{q})[I_k - \mathbf{q} \cdot \nabla_{\mathbf{k}} I_k + \mathcal{O}(\mathbf{q}^2)]. \tag{19}$$

Inserting the above expression into Eq. (17), we get to the leading order

$$\ddot{A}_{k,\alpha} + (k^2 \pm k\dot{I}_k) A_{k,\alpha} \simeq 0, \tag{20}$$

where we used $\Upsilon_{\mathbf{k},\alpha;\,\mathbf{k},\beta} = \pm k \delta_{\alpha\beta}$ and, from now on, $\pm$ refer to $\alpha = 1, 2$, respectively.

In the next three paragraphs, we will analyze the generation of magnetic fields from the vacuum considering a simple phenomenological form for the function I_k. In Section II e, instead, we will give two examples of particle physics models in which that form could naturally arise.

b. A Simple Toy Model

Let us suppose that I_k follows a power-law behavior:

$$I_k = \frac{g}{(-k\eta)^\beta}\,, \tag{21}$$

where g is a dimensionless quantity (which for definiteness we take to be positive) and we assume that $\beta > 0$. [We note that the case analyzed by Prokopec in Ref. [11] corresponds to take $\beta \to 0$ and $g\beta = \text{const} \neq 0$ in Eq. (21)].

For $|k\eta| \ll 1$ we can neglect the term proportional to k^2 in Eq. (20), so that its solution is [1]

$$\beta \neq 1: \; A_{k,\alpha}(\eta) = |k\eta|^{1/2}\Big[c_1 H^{(1)}_{1/(1-\beta)}(z) + c_2 H^{(2)}_{1/(1-\beta)}(z)\Big], \tag{22}$$

where

$$z = \frac{2\sqrt{\pm\beta g}}{1-\beta}\,|k\eta|^{(1-\beta)/2}, \tag{23}$$

c_1 and c_2 are constants of integration, and $H^{(1)}_\nu(x)$ and $H^{(2)}_\nu(x)$ are the Hankel functions of first and second kind, respectively. The case $\beta = 1$ will be analyzed separately (see below).

Since $z \propto |k\eta|^{(1-\beta)/2}$, for $|k\eta| \ll 1$ we have two different cases: if $0 < \beta < 1$ then $|z| \ll 1$, while if $\beta > 1$ then $|z| \gg 1$. Consequently, in Eq. (22) we can use the asymptotic expansion of the Hankel functions for small and large arguments, respectively:

$$H^{(1,2)}_\nu(x) \simeq \begin{cases} \mp\frac{i2^\nu \csc(\pi\nu)}{\Gamma(1-\nu)}\, x^{-\nu} + \frac{2^{-\nu}[1\pm i\cot(\pi\nu)]}{\Gamma(1+\nu)}\, x^\nu, & |x| \ll 1, \nu \neq 0, \\ \pm 2i\pi^{-1}\ln x, & |x| \ll 1, \nu = 0, \\ \sqrt{\frac{2}{\pi x}}\, e^{\pm i(x - \frac{\pi}{4} - \frac{\pi\nu}{2})}, & |x| \gg 1. \end{cases} \tag{24}$$

If $0 < \beta < 1$, using the first equation of (24), we find $A_{k,\alpha}(\eta) \simeq c_1' + c_2'|k\eta|$, where c_1' and c_2' are constants. Since for modes well inside the horizon ($|k\eta| \to \infty$) we have the plane-wave solution $A_{k,\alpha}(\eta) = e^{ik\eta}$ (the normalization corresponds to the standard Bunch-Davies vacuum [27]), and for modes well outside the horizon we have the above solution, we can fix the values of c_1' and c_2' by matching the two solutions and their first derivatives at the

[1] The correct condition in order to neglect the term proportional to k^2 in Eq. (20) is: $|k\eta| \ll (\beta g)^{1/(\beta+1)}$. However, neglecting mathematical quibbles such as the case where $g \to 0$ or $\beta \to 0$, we assume that both β and g are quantities of order unity, so the above relation reads $|k\eta| \ll 1$.

horizon crossing, $|k\eta| = 1$. We find $c'_1 = (1+i)e^{-i}$ and $c'_2 = -ie^{-i}$, so as a final result we can write the expression for the electromagnetic field during inflation and at large scales: [2]

$$0 < \beta < 1: \; A_{k,\alpha}(\eta) \simeq e^{-i}\,[\,1 + i(1 - |k\eta|)\,]. \tag{25}$$

If $\beta > 1$, using the third equation of (24), we find $A_{k,\alpha}(\eta) \simeq |k\eta|^{(1+\beta)/2}(c''_1 e^{iz} + c''_2 e^{-iz})$, where c''_1 and c''_2 are constants. We see that the positive helical modes [corresponding to take the plus sign in Eq. (23)] are $A_{k,+}(\eta) \propto |k\eta|^{(1+\beta)/2}$ (we neglected inessential oscillating factors), while the negative ones are $A_{k,-}(\eta) \propto |k\eta|^{(1+\beta)/2} e^{|z|}$. Therefore, the spectrum of positive helicity states is vanishingly small, while that of negative helicity states presents an infrared divergence (associated to the exponential factor). Therefore, the case $\beta > 1$ is meaningless and then will be neglected in the following.

In the case $\beta = 1$, Eq. (20) can be solved exactly (for all $|k\eta|$):

$$\beta = 1: \; A_{k,\alpha}(\eta) = \sqrt{\frac{\pi}{2}}\, e^{-i\pi(1+2\nu)/4}\, |k\eta|^{1/2}\, H_\nu^{(2)}(|k\eta|), \tag{26}$$

where

$$\nu = \sqrt{\frac{1}{4} \mp g} \tag{27}$$

and we used the normalization corresponding to the Bunch-Davies vacuum.

For large scales, we can replace the Hankel function with its small-argument expansion in Eq. (26).

c. Electromagnetic Backreaction on Inflation

Before proceeding further, we want to analyze the problem of backreaction of the inflation-produced electromagnetic field on the dynamics of inflation. [3] Since in the following we will

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[2] It is worth noting that the matching procedure we have adopted can give us only approximate results. Nevertheless, we believe that the simplified analysis performed in the case $0 < \beta < 1$ catches the main characteristics of the process of creation of magnetic field during inflation.

[3] After inflation, since the conductivity of the cosmic plasma becomes very high and, consequently, the electric field is washed out [3], the electromagnetic energy is simply given by the magnetic one. Moreover, the magnetic field evolves adiabatically from the end of inflation until today, and its energy density is always subdominant with respect to the energy density of the universe (strictly speaking, this is true for cosmic magnetic fields whose intensity is consistent with astrophysical observations, that is $B \lesssim 10^{-9}$G). This means that we can safely neglect the backreaction of the electromagnetic field on the standard evolution of the universe from inflation until today.

assume that the electromagnetic field does not appreciably perturb the standard evolution of the universe, we must verify that the electromagnetic energy density is smaller than the energy density associated to inflation which, during de Sitter inflation, is a constant: $\rho_{\rm infl} = T_1^4$. Here, T_1 is the so-called reheating temperature, that is the temperature of the cosmic plasma at the beginning of the radiation era (here and in the following we assume that the reheating phase, during which the energy of the inflaton is converted into ordinary matter is "instantaneous" so that, after inflation, the universe enters directly the radiation era).

Starting from the definition of the energy-momentum tensor,

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \, , \tag{28}$$

where $S = \int d^4x \sqrt{-g}\, \mathcal{L}$ is the action, and taking into account Lagrangian (1), we find

$$T_{\mu\nu} = \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - F_{\mu}^{\ \ \alpha} F_{\nu\alpha}, \tag{29}$$

that is the standard Mawxell energy-momentum tensor for the electromagnetic field. Here, we have assumed, for the sake of simplicity, that the quantity I does not explicitly depend on the metric tensor $g_{\mu\nu}$ (two particle physics models in which I displays this property are discussed in Section II e). The electromagnetic energy density, $\rho = T_0^0$, is then given by

$$\rho = \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2), \tag{30}$$

where $a^2 \mathbf{B} = \nabla \times \mathbf{A}$ and $a^2 \mathbf{E} = -\dot{\mathbf{A}}$. Expanding the electromagnetic field as in Eqs. (3)-(4), replacing $\mathbf{B}_\alpha$ and $\mathbf{E}_\alpha$ with their average values $\mathbf{B}_{\lambda,\alpha}$ and $\mathbf{E}_{\lambda,\alpha}$, and then taking the vacuum expectation value of expression (30), we obtain the vacuum expectation value of the average electromagnetic energy density on a comoving scale λ:

$$\rho_\lambda = \frac{1}{2} \sum_{\alpha=1,2} \left(B_{\lambda,\alpha}^2 + E_{\lambda,\alpha}^2 \right) . \tag{31}$$

In order to have a self-consistent model for the production of a cosmic magnetic field, we must verify that, on the scale of interest λ, the electromagnetic energy produced during inflation is less than the total energy of the universe:

$$\rho_\lambda < \rho_{\rm infl}. \tag{32}$$

Taking into account Eqs. (7)-(12) and Eqs. (25), (26), and (31), we easily find, at large scales:

$$0 < \beta < 1: \; \rho_\lambda(\eta) \simeq \frac{3H^4}{8\pi^2} \left(\frac{\eta}{\lambda}\right)^4 \tag{33}$$

and

$$\beta = 1: \; \rho_\lambda(\eta) \simeq \frac{H^4}{32\pi^3} \, 4^{\nu_+} \left(\frac{1}{2} - \nu_+\right)^2 [\Gamma(\nu_+)]^2 \, \Gamma\!\left(\frac{3}{2} - \nu_+\right) \left(\frac{|\eta|}{\lambda}\right)^{3-2\nu_+}, \tag{34}$$

where $\nu_+ = \sqrt{\frac{1}{4} + g}$. It is important to observe that, in order to avoid an infrared divergence in the magnetic power spectrum, we must impose that $\nu < 3/2$.

Taking into account that $T_1^4 \simeq H^2 m_{\text{Pl}}^2$ [3], where $m_{\text{Pl}} \simeq 1.22 \times 10^{19}\text{GeV}$ is the Planck mass, we have approximatively

$$0 < \beta < 1: \; \frac{\rho_\lambda}{\rho_{\text{infl}}} \sim 10^{-10} \left(\frac{T_1}{10^{-2} m_{\text{Pl}}}\right)^4 \left(\frac{|\eta|}{\lambda}\right)^4 \tag{35}$$

and

$$\beta = 1: \; \frac{\rho_\lambda}{\rho_{\text{infl}}} \sim 10^{-10} \frac{f(\nu_+)}{3 - 2\nu_+} \left(\frac{T_1}{10^{-2} m_{\text{Pl}}}\right)^4 \left(\frac{|\eta|}{\lambda}\right)^{3-2\nu_+}, \tag{36}$$

where $f(x)$ is an increasing function of x, such that $f(1/2) = 0$ and $f(3/2) \sim 1$. Since analyses of CMB radiation and Big Bang Nucleosynthesis constrain the reheating temperature in the range $1\text{GeV} \lesssim T_1 \lesssim 10^{-2} m_{\text{Pl}}$ [3], and since we are considering the case of large scales, $|\eta|/\lambda \ll 1$, we see form Eqs. (35) and (36) that we can safely neglect the backreaction of the electromagnetic field on the dynamics of inflation.

d. The Actual, Inflation-produced, Magnetic Field

After inflation, the universe enters the radiation era (assuming instantaneous reheating). We restrict our analysis to the case in which, during this era, the interaction term in Lagrangian (1) is (for some reason) negligible, so that the general expression for the electromagnetic field is

$$A_{k,\alpha}^{\text{rad}}(\eta) = \alpha_{k,\alpha} e^{ik\eta} + \beta_{k,\alpha} e^{-ik\eta}. \tag{37}$$

Here, $\alpha_{k,\alpha}$ and $\beta_{k,\alpha}$ are the so-called Bogoliubov coefficients [27], determining the spectral number distribution of particles produced from the vacuum. By matching expressions (25) and (37) and their first derivatives at the time of the end of inflation, $\eta = \eta_1$, we find the spectrum of the electromagnetic field generated from the vacuum at large scales:

$$0 < \beta < 1: \; |A_{k,\alpha}^{\text{vac}}(\eta_1)|^2 = |\beta_{k,\alpha}|^2 \simeq \frac{1}{4}\,. \tag{38}$$

In the same way, we find:

$$\beta = 1:\ |A_{k,\alpha}^{\rm vac}(\eta_1)|^2 \simeq \begin{cases} \frac{[2^\nu\left(\frac12-\nu\right)\Gamma(\nu)]^2}{8\pi|k\eta_1|^{1+2\nu}}, & \nu > 0, \\ \frac{(\ln|k\eta_1|)^2}{8\pi|k\eta_1|}, & \nu = 0, \\ \frac{a_\nu + b_\nu \cos[2\tilde{\nu}\ln(|k\eta_1|/2)]}{4\pi|k\eta_1|} + \frac{c_\nu \sin[2\tilde{\nu}\ln(|k\eta_1|/2)]}{4\pi|k\eta_1|}, & \nu = i\tilde{\nu}. \end{cases} \tag{39}$$

Here, $\tilde{\nu} > 0$ is the imaginary part of ν and

$$\begin{aligned} a_\nu &= \frac{\pi}{\tilde{\nu}}\,(1+4\tilde{\nu}^2)\coth(\pi\tilde{\nu}), \\ b_\nu &= (1-4\tilde{\nu}^2){\rm Re}[\Gamma(\nu)^2] + 4\tilde{\nu}{\rm Im}[\Gamma(\nu)^2], \\ c_\nu &= (1-4\tilde{\nu}^2){\rm Im}[\Gamma(\nu)^2] - 4\tilde{\nu}{\rm Re}[\Gamma(\nu)^2], \end{aligned} \tag{40}$$

where ${\rm Re}[x]$ and ${\rm Im}[x]$ are the real and imaginary part of x.

We observe that when $g = 0$ ($\nu = 1/2$) conformal invariance is recovered and then, as it should be, we find $|A_{k,\alpha}^{\rm vac}(\eta_1)|^2 = 0$.

From the end of inflation until today, due to the high conductivity of the cosmic plasma, the magnetic field evolves adiabatically [3], $a^2 B_{\lambda,\alpha} = $ const, so that $B_{\lambda,\alpha}^{\rm today} = a_1^2 B_{\lambda,\alpha}(\eta_1)$. Inserting Eqs. (38) and (39) in Eq. (9), we obtain respectively

$$0 < \beta < 1:\ B_{\lambda,\alpha}^{\rm today} \simeq \frac{1}{4\sqrt{2}\pi\lambda^2}\,, \tag{41}$$

and

$$\beta = 1:\ B_{\lambda,\alpha}^{\rm today} \simeq \begin{cases} \frac{2^\nu|\frac12-\nu|\,\Gamma(\nu)\,[\Gamma(\frac32-\nu)]^{1/2}}{(4\pi)^{3/2}\lambda^2}\left(\frac{\lambda}{|\eta_1|}\right)^{\nu+1/2} & \nu > 0, \\ \frac{\ln(\lambda/|\eta_1|)}{2(4\pi)^{5/4}\lambda^2}\left(\frac{\lambda}{|\eta_1|}\right)^{1/2} & \nu = 0, \\ \frac{(a_\nu + b_\nu/\sqrt{2} + c_\nu/\sqrt{2})^{1/2}}{\sqrt{2}(4\pi)^{5/4}\lambda^2}\left(\frac{\lambda}{|\eta_1|}\right)^{1/2} & \nu = i\tilde{\nu}, \end{cases} \tag{42}$$

where, for simplicity, we replaced in the third equation of (39) the cosine and sine functions with their root-mean-square values $1/\sqrt{2}$.

It is useful to observe that

$$\frac{\lambda}{|\eta_1|} \simeq 1.22\times 10^{23}\,\frac{T_1}{m_{\rm Pl}}\,\lambda_{\rm 10kpc}, \tag{43}$$

$$\lambda^{-2} \simeq 0.60\times 10^{-53}\lambda_{\rm 10kpc}^{-2}{\rm G}, \tag{44}$$

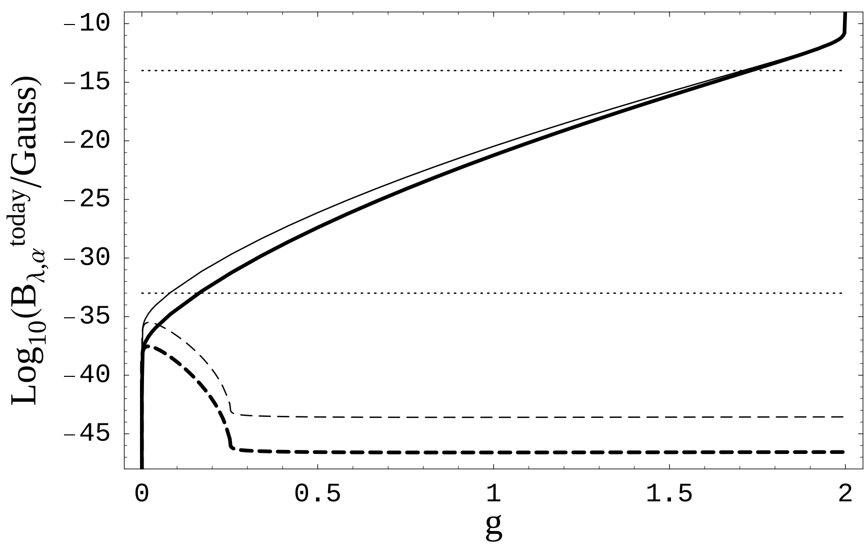

FIG. 1: Actual magnetic field in the case $\beta = 1$ as a function of g at the comoving scales $\lambda = 10$kpc (thin lines) and $\lambda = 1$Mpc (thick lines), for a reheating temperature $T_1 = 10^{-2} m_{\rm Pl}$. Dashed and continuous lines refer to the cases $\alpha = 1$ (positive helicity states) and $\alpha = 2$ (negative helicity states), respectively. The horizontal dotted lines refer to the minimum seed fields required for dynamo amplification, $B \simeq 10^{-33}$G, and to directly explain galactic magnetism, $B \simeq 10^{-14}$G.

where $\lambda_{\rm 10kpc} = \lambda/(10{\rm kpc})$. In Eq. (43), we used the fact that during radiation and matter dominated eras the expansion parameter evolves as $a \propto g_{*S}^{-1/3} T^{-1}$, where T is the temperature and $g_{*S}(T)$ the number of effectively massless degrees of freedom referring to the entropy density of the universe [28]. [4]

Taking into account Eqs. (41) and (43)-(44), we see that magnetic fields produced in the cases $0 < \beta < 1$ are well below the minimum seed field required for dynamo amplification, $B \simeq 10^{-33}$G.

In Fig. 1, we show the actual magnetic field in the case $\beta = 1$ as a function of the parameter g at the comoving scales $\lambda = 10$kpc (thin lines) and $\lambda = 1$Mpc (thick lines), for $T_1 = 10^{-2} m_{\rm Pl}$. Dashed and continuous lines refer to the cases $\alpha = 1$ (positive helicity states) and $\alpha = 2$ (negative helicity states), respectively.

As it is clear from the figure, astrophysically interesting fields can be produced as exci-

[4] In this paper, we use the values [28]: $T_0 \simeq 2.35 \times 10^{-13}$GeV, $g_{*S}(T_0) \simeq 3.91$, and $g_{*S}(T_1) = 106.75$ (referring to the number of effectively massless degrees of freedom of Standard Model). It is useful to know that $1{\rm G} \simeq 6.9 \times 10^{-20}{\rm GeV}^2$ and $1{\rm Mpc} \simeq 1.56 \times 10^{38}{\rm GeV}^{-1}$.

tation of the vacuum. They have a definite helicity [5] and intensity strongly depending on the value of g. (The case $g \geq 2$, or equivalently $\nu > 3/2$, corresponds to a non-physical infrared-divergent magnetic power spectrum). Numerically we find that for $g \gtrsim 0.1$ the produced field on scales of 10kpc is stronger than the minimum seed field required for a successful galactic dynamo amplification, while if $1.7 \lesssim g < 2$ its intensity on scales of 1Mpc is high enough to directly explain galactic magnetism ($B \gtrsim 10^{-14}$G). In the range of interest, $0.1 \lesssim g \lesssim 2$, we have approximately

$$B_{\lambda,-}^{\rm today} \sim \; 10^{23\nu-47} \left(\frac{T_1}{10^{-2} m_{\rm Pl}} \right)^{\nu+1/2} \lambda_{10{\rm kpc}}^{\nu-3/2} \; {\rm G}, \tag{45}$$

where we used the first equation of (42) with $\alpha = 2$, and Eqs. (43)-(44).

e. Examples of Particle Physics Models

We want now give two particle physics models in which the function $I_{\mathbf{k}}$ takes on the form (18) with I_k given by Eq. (21).

Let us start by considering a model in which I is given by $I = \phi/M$, where M is a mass scale and ϕ a pseudoscalar (background) field whose Lagrangian density is

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + V(\phi), \tag{46}$$

with $V(\phi)$ a potential term that will be specified later.

In the spirit of mean field theory, we may approximate the function $\phi(\mathbf{x}, \eta)$ by its root-mean-square value $\phi_\lambda(\eta)$ on the scale λ:

$$\phi_\lambda^2(\eta) = \langle 0 | \, | \int d^3y W_\lambda(|\mathbf{x} - \mathbf{y}|) \phi(\mathbf{y}, \eta)|^2 \, |0\rangle, \tag{47}$$

where $W_\lambda(|\mathbf{x}|)$ is a suitable window function. Introducing the ϕ-power spectrum, $\mathcal{P}_\phi(k, \eta)$, through the relation

$$\langle 0 | \phi_{\mathbf{k}}(\eta) \phi_{\mathbf{q}}^*(\eta) | 0 \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{q}) \frac{2\pi^2}{k^3} \mathcal{P}_\phi(k, \eta), \tag{48}$$

where $\phi_{\mathbf{k}}$ is the Fourier transform of $\phi(\mathbf{x})$, it is straightforward to obtain

$$\phi_\lambda^2(\eta) = \int_0^\infty \frac{dk}{k} \, |W_\lambda(k)|^2 \, \mathcal{P}_\phi(k, \eta), \tag{49}$$

[5] Since we are considering $g \geq 0$, only negative helicity states are astrophysically relevant. Had we taken $g \leq 0$, the non-vanishing modes would have been the positive ones.

where $W_\lambda(k)$ is the Fourier transform of the window function. For the sake of simplicity, let us assume that $|W_\lambda(k)|^2$ picks out just modes with wavenumber $k = 1/\lambda$. In other words, we take $|W_\lambda(k)|^2 = \delta(k\lambda - 1)$, where $\delta(x)$ is the Dirac delta function. Then, we may write: $\phi^2_\lambda(\eta) = \mathcal{P}_\phi(1/\lambda, \eta)$. Consequently, the Fourier transform of $I(\mathbf{x}, \eta)$ is simply given by:

$$I_{\mathbf{k}}(\eta) = (2\pi)^3 \delta^3(\mathbf{k}) \, \frac{[\mathcal{P}_\phi(1/\lambda, \eta)]^{1/2}}{M} \, . \tag{50}$$

To proceed further, we assume that the ϕ-power spectrum is peaked at large scales, $\lambda \to \infty$. Taking into account the presence of the Dirac function in Eq. (50), we can roughly write $I_{\mathbf{k}}(\eta) \simeq (2\pi)^3 \delta^3(\mathbf{k}) [\mathcal{P}_\phi(k, \eta)]^{1/2}/M$, since both k and $1/\lambda$ go to zero. Therefore, the function I_k introduced in Eq. (18) takes on the form

$$I_k(\eta) \simeq \frac{[\mathcal{P}_\phi(k, \eta)]^{1/2}}{M} \, . \tag{51}$$

To be more concrete, let us now consider the case in which ϕ is either a massive pseudoscalar field minimally coupled to gravity and a massless pseudoscalar field non-minimally coupled to gravity. They are described by Lagrangian (46) with

$$V(\phi) = \begin{cases} m^2 \phi^2, \\ \xi R \phi^2, \end{cases} \tag{52}$$

respectively. Here, m^2 is the squared mass of the pseudoscalar field, ξ a real parameter, and R the Ricci scalar. It is well-known that in those cases, the ϕ-power spectrum in de Sitter background is given by (see, e.g., Ref. [29]):

$$\mathcal{P}_\phi(k, \eta) = \left(\frac{H}{2\pi} \right)^2 (-k\eta)^{3 - 2\nu_\phi}, \tag{53}$$

where

$$\nu_\phi = \begin{cases} \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \, , \\ \sqrt{\frac{9}{4} - 12 \, \xi} \, , \end{cases} \tag{54}$$

in the two cases, respectively. Comparing Eq. (51) with Eq. (21), we finally get

$$g = \frac{H}{2\pi M} \, , \tag{55}$$

$$\beta = \nu_\phi - \frac{3}{2} \, . \tag{56}$$

It is clear that only negative values of m^2 and ξ give positive values of β. Moreover, the astrophysically interesting case discussed in the previous Section, $\beta = 1$ and $0.1 \lesssim g \lesssim 2$, corresponds to have $m^2 = -4H^2$ and $\xi = -1/3$, and $0.1H \lesssim M \lesssim 1.6H$.

III. CREATION OF MAGNETIC HELICITY

Let us now consider the creation of magnetic helicity in our model. The magnetic helicity in a volume $V = \lambda^3$ can be conveniently defined as

$$H_\lambda(\eta) = \langle 0| \int d^3y \int d^3z W_\lambda(|\mathbf{x}-\mathbf{y}|) W_\lambda(|\mathbf{x}-\mathbf{z}|) \mathbf{A}(\eta,\mathbf{y}) \cdot \mathbf{B}(\eta,\mathbf{z}) |0\rangle, \tag{57}$$

where $W_\lambda(|\mathbf{x}|)$ is a gaussian window function. Taking into account Eqs. (3)-(4), we obtain

$$H_\lambda(\eta) = \int_0^\infty \frac{dk}{k} \, W_\lambda^2(k) \, \mathcal{H}_k(\eta), \tag{58}$$

where

$$\mathcal{H}_k(\eta) = \frac{k^3}{4\pi^2 a^2} \left[\, |A_{k,+}(\eta)|^2 - |A_{k,-}(\eta)|^2 \, \right] \tag{59}$$

is the magnetic helicity power spectrum (which is well defined only when $\mathcal{H}_k/k$ is integrable in $k \to 0$).

A magnetic field is said to be "maximally helical" at the time $\bar{\eta}$ if either $A_{k,+}(\bar{\eta})$ or $A_{k,-}(\bar{\eta})$ is zero.

Looking at Eqs. (38) and (59), we immediately get that in the case $0 < \beta < 1$ the magnetic helicity is zero. In the case $\beta = 1$, instead, taking into account Eq. (39), we obtain

$$H_\lambda^{\rm today} \simeq \begin{cases} \frac{\gamma_+}{\sqrt{\pi}} \, (B_{\lambda,+}^{\rm today})^2 \lambda - \frac{\gamma_-}{\sqrt{\pi}} \, (B_{\lambda,-}^{\rm today})^2 \lambda, \;\; 0 \leq g \leq \frac{1}{4}, \\ \\ \frac{2}{\sqrt{\pi}} \, (B_{\lambda,+}^{\rm today})^2 \lambda - \frac{\gamma_-}{\sqrt{\pi}} \, (B_{\lambda,-}^{\rm today})^2 \lambda, \;\; \frac{1}{4} \leq g < \frac{3}{4}, \end{cases} \tag{60}$$

where $\gamma_\pm = \mathrm{B}(1/2\,, 1-\nu)$ and $\mathrm{B}(x,y)$ is the Euler beta function. (The case $g \geq 3/4$ corresponds to an infrared-divergent magnetic helicity power spectrum.)

It is worth noting that, al least in the case of astrophysical interest, the generated magnetic fields possess a definite helicity (in particular they are left-handed since $|A_{k,+}| \ll |A_{k,-}|$). Hence, they are (almost) maximally helical, and the amount of created magnetic helicity is roughly given by

$$H_\lambda^{\rm today} \sim -(B_{\lambda,-}^{\rm today})^2 \lambda. \tag{61}$$

A helical magnetic field leaves peculiar imprints on the CMB radiation. Unfortunately, the maximal helicity producible in our mechanism is much smaller than that detectable in near future CMB experiments which is, in the most optimistic case, of order of $(10^{-9}\mathrm{G})^2\mathrm{Mpc}$

on Mpc scales [30]. In fact, at those scales we get from Eq. (61) and Fig. 1 the value $|H_\lambda^{\rm today}|_{\rm max} \sim (10^{-24}{\rm G})^2{\rm Mpc}$.

Interesting enough, it has been showed by Kahniashvili and Vachaspati [31] that, in principle, the study of propagation properties of charged cosmic rays through a magnetic field could give information about the helicity of the field itself.

IV. CONCLUSIONS

Up to today, the hypothesis that *all* galaxies are pervaded by microgauss magnetic fields has not been ruled out yet. Due to their large correlation length, we can entertain the idea that they are remnant of an inflationary epoch of the universe.

In this paper, we have indeed analyzed the production of seed magnetic fields during de Sitter inflation. We have considered a photon interaction term in the electromagnetic Lagrangian (which breaks conformal invariance of electrodynamics) of the form $-\frac{1}{4}IF_{\mu\nu}\widetilde{F}^{\mu\nu}$, where I is a pseudoscalar function. We have considered a simplified special case where I is peaked at large scales ($k = 1/\lambda \rightarrow 0$) during inflation and then is vanishingly small afterwards. In particular, we have studied the case where I is parameterized by the power-law behavior $I(k,\eta) = g/(-k\eta)^\beta$, with g a positive dimensionless constant, β a real positive number, and η the conformal time. Also, we have shown that this particular form of I could be realized owing to the coupling between the photon and either a (tachyonic) massive pseudoscalar field and a massless pseudoscalar field non-minimally coupled to gravity.

We have found that only when $\beta = 1$ astrophysically interesting fields can be produced as excitation of the vacuum. They are maximally helical and have an intensity strongly depending on the value of g (which, in order to avoid an infrared divergence in the magnetic power spectrum, is constrained in the range $0 \leq g < 2$). In particular, for $g \gtrsim 0.1$ the produced fields on scales of 10kpc are stronger than the minimum seed field required for a successful galactic dynamo amplification ($B \gtrsim 10^{-33}$G), while if $1.7 \lesssim g < 2$ their intensity on scales of 1Mpc is high enough to directly explain galactic magnetism ($B \gtrsim 10^{-14}$G).

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